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Journal of Computational and Applied Mathematics 179 (2005) xv–xix

JOURNAL OF  
COMPUTATIONAL AND  
APPLIED MATHEMATICS[www.elsevier.com/locate/cam](http://www.elsevier.com/locate/cam)

# Biography of Olav Njåstad

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Olav Njåstad was born on March 1, 1933 in Osterøy, an island in the western part of Norway, near Bergen, where he also grew up. His parents were farmers in combination with craft activity, a rather common way of making one's living at that time.

After elementary school he went to Bergen for high school. In those days this was rather exceptional for young people from the countryside, but Olav showed strong interest in intellectual pursuits, and he had the will to follow his inclinations. He graduated from Hordaland Gymnas in 1953, and was anxious to start on studies in science and mathematics, but had to do his mandatory military service first. He solved this problem by starting his studies simultaneously. His subjects were mathematics, astronomy, mechanics and statistics. Already then, and ever since, he had in addition a strong interest in nature, hiking, cross-country skiing and history.

In 1958 he finished his cand.real.-degree from The University of Oslo, majoring in mathematics. His thesis dealt with the Peano axioms, and was carried out very independently. The main results were later published in an article.

Olav's first position was as a high school teacher (lektor) at Riis school in Oslo, 1958–1959. In addition, he was also teaching at the University. During his stay in Oslo he got acquainted with Laila Løvseth, who was in the process of finishing her studies as a pharmacist. They married in 1958. Their children, Oddvar, Magne and Birgit were born in 1959, 1962, and 1966, respectively.

In 1959 Olav got a position as universitetslektor (assistant professor) at The University of Oslo, a position he held until he, in 1966, was appointed dosent (associate professor) at the University of Trondheim. In 1972 he was appointed full professor. See [Fig. 1](#) (a) and (b).

During the initial years Olav mostly worked on general topology. In the start he was inspired by his colleagues Erik Alfsen and Jens Erik Fenstad, but soon went on independently and published several papers throughout the years with important results. The papers are generally very well written. They are known to have contributed substantially to a satisfactory theory for uniformity, proximity and the associated compactification problem. His studies in general topology resulted in his thesis "Uniformity, proximity and compactification," for which he was granted the degree dr.philos. in 1967.

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Fig. 1.

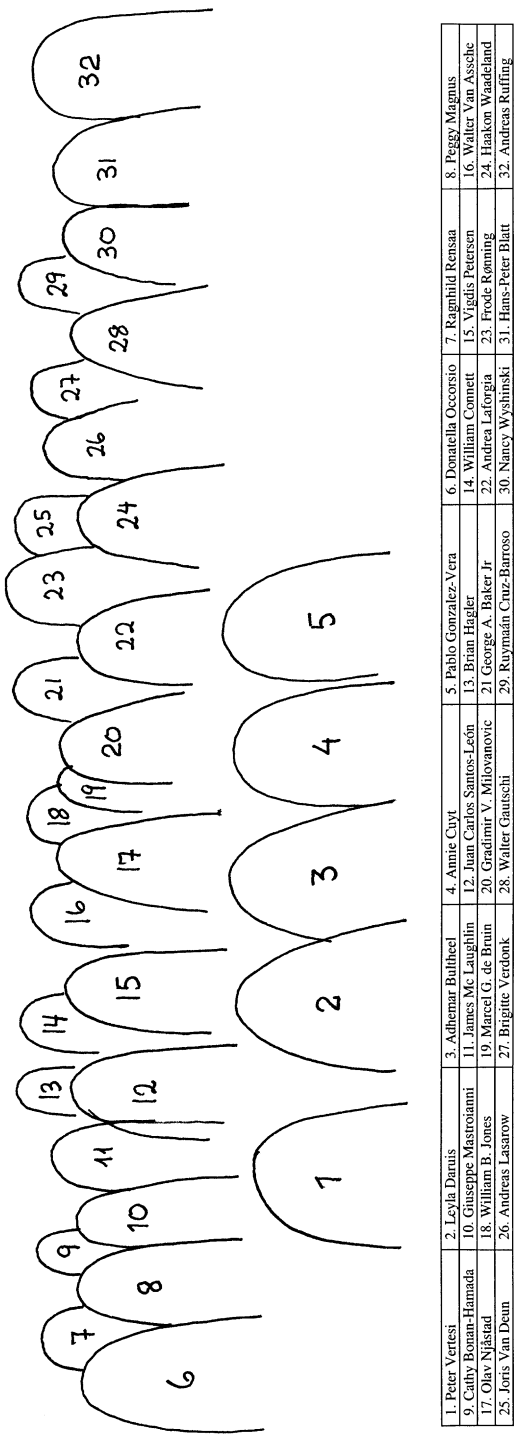


Fig. 1. (Continued.)

Olav Njåstad gradually expanded his field of research. He studied boundary values of holomorphic functions. A theorem of Bagemihl–Seidel was an inspiration and a starting point for two papers where he, by a large number of results, contributed to a technically very complicated area.

Olav went on in his research, now to function algebras, with main emphasis on the approximation theory in the sense of Mergelyan and Bishop. He introduced a generalized peak point and extended a theorem of Bishop on conditions for certain rational functions on a set  $X$  with poles outside of  $X$  to be dense in the family of continuous functions on  $X$ . In a later paper he was able to prove other results under weaker conditions than was known earlier. One such example is the Walsh–Lebesgue theorem.

In his position at The University of Oslo and more so later at The University of Trondheim he had a wide scope of teaching, from elementary courses to supervising of thesis and to research seminars. He is known for his well prepared, precise and careful presentation and is highly rated among his students and colleagues. He has supervised 15 students at cand.real- and siv.ing. level and 2 Ph. D. students. Moreover, he has four times been visiting professor at The University of Colorado at Boulder, starting 1973–74.

Being highly talented also in areas other than mathematics Olav had to take his share and more of administrative duties, all of which he has carried out in an excellent way. He has been Chairman of The Department of Mathematical Sciences, Dean of The Faculty of Physics and Mathematics, Substitute member of The Research Council for Science and the Humanities and Chairman of The Norwegian Council of Mathematics. He also served on the very important Central Budget Committee at the University.

Olav Njåstad has served on a large number of committees for doctoral thesis and for evaluation of applicants for positions. He has acted as a reviewer for Mathematical Reviews on a large number of papers and has also been a referee for many different journals.

Olav Njåstad is a member of The Royal Norwegian Society of Science and Letters and The Norwegian Academy of Technical Sciences.

Since the late 1950s the Department of Mathematics in Trondheim has had a very close contact with the University of Colorado. A mutual exchange of people took place rather frequently. This resulted in joint seminars and joint papers. Olav soon became a participant in this exchange and turned gradually into an essential partner in the cooperation. Of importance, in particular in the beginning, was the contact with Wolfgang J. Thron, who had general topology as one of his research areas, in addition to analytic theory of continued fractions. He, and some of his students, were substantial in the contact. Olav and Wolf collaborated closely on problems in general topology and soon also on continued fractions.

Since the early 1980s, the Boulder connection brought along with continued fractions also Padé approximation, moment problems, orthogonal polynomials, and all their generalizations into the titles of Olav Njåstad's publications. A few words about the area Olav entered and gradually has contributed essentially to: a classical moment problem which can be described as follows: given a sequence  $\{c_n\}_{n \in \mathbb{N}}$  of numbers, find conditions under which there exists a (positive) measure  $\mu$  with moments  $m_n = \int t^n d\mu(t)$  that satisfies  $c_n = m_n$  for  $n \in \mathbb{N}$ . If a solution exists, what are the conditions for uniqueness, and if the solution is not unique, describe all the solutions. For the classical Hamburger and Stieltjes moment problems, the support being  $\mathbb{R}$  and  $\mathbb{R}_0^+$ , respectively, the sequence  $\{c_n\}$  is real and  $N = \mathbb{N}_0$ . In solving these problems, orthogonal polynomials are crucial. Their zeros are used to construct Gauss-type quadrature formulas, which correspond to a discrete measure that under appropriate conditions will converge to a solution of the moment problem. These orthogonal polynomials also are denominators of rational approximants to the Stieltjes transforms  $\hat{\mu}(z) = \int (t - z)^{-1} d\mu(t)$ . This rational approximation is in a Padé sense since a maximal number of initial moments is matched, giving maximal contact at infinity. The orthogonal polynomials also satisfy a 3-term recurrence relation, which is equivalent to a continued fraction. This

briefly catches the circle of ideas entangled with almost all of Olav's publications since 1983, but has also been an inspiration to other problems.

The earliest contributions of Olav Njåstad to this field are generalizations of these problems in the case where it is prescribed to match the complete two-sided sequence of moments, i.e.  $N = \mathbb{Z}$  in the previous setting. These are then called *strong* moment problems. Since then, in the definition of the moments positive as well as negative powers are used, the orthogonal polynomials have to be replaced by orthogonal Laurent polynomials, and the Stieltjes transforms are approximated at two points simultaneously, namely at 0 and  $\infty$ . So Padé approximation is replaced by two-point Padé approximation. All the aspects of the moment problems were treated in many papers by Olav Njåstad ranging from 1983 to 2003. Several of these were in collaboration with W.B. Jones and W.J. Thron, and occasionally also with H. Waadeland. He also went on to the trigonometric moment problem, where there is a somewhat different situation. He elaborated the theory of continued fraction in this context. He collaborated with W. Thron and W.B. Jones. A highlight of this cooperation is a paper from 1989 in Bull. Lond. Math. Soc., which has been a great help for people working in this area. It is heavily cited. He has also contributed to frequency analysis problems in collaboration with W.B. Jones, W.J. Thron and H. Waadeland.

Since 1985 a collaboration with P. González-Vera (La Laguna) and E. Hendriksen (Amsterdam) was started. They shared (with O. Njåstad) an interest in the above-mentioned problems originally concerning (two-point) Padé approximation, quadrature formulas and strong moment problems. But soon it was realized that the extension from one to two point approximation was just one step of a generalization to multipoint approximation. The moment problems were coined *extended* moment problems by O. Njåstad, and here both the moment problems on the line and on the circle could be generalized. A whole theory had to be established, orthogonal polynomial had to be replaced by orthogonal rational functions, to mention but one example. In the first papers on multipoint problems there were only a finite number of poles, cyclically repeated. After A. Bultheel (Leuven) joined in 1989 a general sequence of poles were considered. Although these orthogonal rational functions were introduced already in the 1960s by A. Djrbashian (Yerevan) a systematic analysis started around 1990 by these authors. The four of them, commonly known to their many friends as “the gang of four”, have been working closely together ever since, resulting in a long list of joint and individual publications. All the aspects related to classical moment problems that were mentioned above, were discussed to some extent in their joint papers from 1991 onwards. A summary of the results was collected in their 1999 monograph, “Orthogonal Rational Functions”. Some special interest has grown in the quadrature aspect as such, loose from the moment problem. This shows in several papers of these authors, but also more classical Szegő quadrature formulas were recently discussed in O. Njåstad's papers with L. Daruis (La Laguna) and W. Van Assche (Leuven). These quadrature rules were used in computations to solve frequency analysis problems mentioned above.

So Olav Njåstad's papers are contributions that help to approximate a beautiful piece of mathematics. They are not restricted to one particular topic, but they are multipoint approximants in every sense of the word. The sequence of his papers is partly orthogonal in the sense of being complementary, however, like orthogonal sequences are also strongly related since a recurrence relation generates the next entry from the previous ones, there is also a thread, a real (red) line, to be found that somehow connects many of them and connects them as a sequence to the same complex circle of ideas.